This Thing Called Flux
(a primer)

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The OxfOrd English Dictionary tells us that flux is synonymous with (though etymologically unrelated to) flow. The earliest documented uses of the word in English, from the 4th Century, were physiological in nature, pertaining specifically to discharge of blood and other bodily fluids or humours. “Flux” in a geohydrological sense (e.g., tidal flux and reflux) came into common use in the 17th Century, and by the late 19th, it had been given its precise modern quantitative physical definition by Maxwell, Tyndall and others.

Maxwell [1891] wrote: “...the velocity of a fluid may be investigated either with respect to the actual velocity of the individual particles, or with respect to the quantity of the fluid which flows through any fixed area” per unit time; the latter is the flux. “But... we require to know separately the density of the body as well as the displacement or velocity, in order to apply the first method, and whenever we attempt to form a molecular theory we have to use the second.” Defined in this way, flux quantifies the average effective velocity of large numbers of infinitesimal fluid elements, whose complex individual motions are in practice both unpredictable and of limited interest.1

Flux depends on the forces driving the flow, the properties of the fluid and those of the medium through which it is flowing. Flux laws are thus constitutive equations, subject to caveats and limits of applicability, rather than universal physical laws like Maxwell’s equations. Darcy’s [1856] Law is a well-known example describing groundwater flow through a saturated porous medium; it relates the groundwater flux q_ [m^3 m^-2 s^-1] to the hydraulic gradient and the hydraulic conductivity. It is identical in form to Fourier’s [1822] Law and to Ohm’s [1827] Law, which respectively describe flow of heat and of electric current. Both heat flow and electric current were historically associated with the motion of metaphysical ‘subtle fluids’: Lavoisier’s caloric and one-fluid (e.g., Franklin) and two-fluid (e.g., Poisson) models of electricity. The similarity in their macroscopic flux laws (Exhibit A, page 7), however, does not require any fundamental similarity of underlying microscopic physical phenomena: “A fluid is certainly a substance, heat is as certainly not a substance, so that though we may find assistance from analogies of this kind in forming clear ideas of formal relations of electrical quantities, we must be careful not to let the one or the other analogy suggest to us that electricity is either a substance like water, or a state of agitation like heat.” [Maxwell, art. 72].

4The electron was discovered in 1897 by J.J. Thomson
**Visiting Fellows’ Reports**

**Magnetic properties of nanofabricated magnetite particle arrays**

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INTERGRAIN MAGNETOSTATIC INTERACTIONS in natural magnetic mineral assemblages have recently attracted much interest due to their potential influence on magnetic properties in general and, more specifically, their influence on determinations of relative and absolute palaeointensity, magnetic granulometry etc. The crux with experimental studies of magnetostatic interactions in rocks is that natural samples display a wide distribution of magnetic mineral composition, grain size, and grain separation and it is thus difficult to separate these various influences on rock magnetic characteristics. Conventional synthetic samples in the form of powders are also only of limited use, as they tend to form clusters and cannot be suitably dispersed especially in the most interesting SD-PSD grain size range.

In order to address this problem, we have resorted to nanofabrication methods commonly used in micro-electronics for fabricating computer chips, memory devices etc. to produce extremely well-defined two-dimensional arrays of uniformly shaped and sized magnetite particles. The fabrication process starts with the sputter deposition of iron on a silicon substrate. Then, electron beam lithography is used to define the particle array. Electron beam lithography uses a focused electron beam to expose arbitrary patterns in an electron sensitive resist layer on top of any two-dimensional substrate. This resist layer can then be developed in the same manner as a photographic film so that only the exposed parts remain. The pattern formed by this process is then transferred into the underlying Fe film by a plasma processing technique called dry etching. Finally, the patterned Fe film is oxidised under controlled oxygen fugacity to be converted into stoichiometric magnetite (details of our fabrication process can be found in Kong et al., in press). This nanofabrication method allows us to not only control the grain size but, crucially, also the grain separation. Figure 1 shows examples of arrays of such particles before (a) and after (b) controlled oxidation.

The aim of my visit to the IRM was to determine the low-temperature remanence properties of those samples in order to confirm the stoichiometry of the magnetite particles and to measure hysteresis properties and FORCs to study how these are affected by magnetostatic interactions. Figure 2a shows low-temperature remanence curves from the sample shown in Figure 1b exhibiting a pronounced Verwey transition and thus confirming the stoichiometry of the magnetite. The FORC diagram for the same sample is shown in Figure 2b. In agreement with the size (250nm) and separation (60nm) of the magnetite particles, the FORC diagram lacks an isolated peak, and has a relatively large vertical spread, indicating magnetostatically interacting PSD-sized particles.

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![Figure 1: SEM images of nanofabricated two-dimensional arrays of Fe particles before oxidation (a) and of magnetite particles after oxidation at controlled oxygen fugacity (b).](image1)

![Figure 2: (a) Low-temperature remanence curves for the sample shown in Figure 1b. The Verwey transition is clearly visible. (b) Room-temperature FORC diagram for the same sample.](image2)
Current Articles

A list of current research articles dealing with various topics in the physics and chemistry of magnetism is a regular feature of the IRM Quarterly. Articles published in familiar geology and geophysics journals are included; special emphasis is given to current articles from physics, chemistry, and materials-science journals. Most abstracts are taken from INSPEC (© Institution of Electrical Engineers), Geophysical Abstracts in Press (© American Geophysical Union), and The Earth and Planetary Express (© Elsevier Science Publishers, B.V.), after which they are subjected to Procrustean culling for this newsletter. An extensive reference list of articles (primarily about rock magnetism, the physics and chemistry of magnetism, and some paleomagnetism) is continually updated at the IRM. This list, with more than 10,000 references, is available free of charge. Your contributions both to the list and to the Abstracts section of the IRM Quarterly are always welcome.

Archeomagnetism

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Magnetic Field Records and Geomagnetism


Magnetization & Demagnetization Processes and Methods


Mineral and Rock Magnetism


Mineral Physics and Chemistry


Nanophase and Disordered Systems


Paleomagnetism and Tectonics


2008 Visiting Fellows, January - June

Dario Bilardello, Lehigh University, Direct measurement of hematite individual particle anisotropy using thermal fluctuation tomography: comparison to other techniques and implications for inclination shallowing in red bed DRMs

Liao Chang, National Oceanography Centre, Southampton, Investigating the grain size dependent magnetic properties of greigite (Fe₃S₄)

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Other


Karin Louzada, Harvard University, Imaging of Experimentally Shocked Pyrrhotite

Nobutatsu Mochizuki, Geological Survey of Japan, Magnetic properties of granitic rocks

Daniel Peppe, Yale University, Magnetic mineralogy of the Cretaceous-Paleocene terrestrial sediments from North Dakota

Chuang Xuan, University of Florida, Rock Magnetic Study on High Latitude Deep Sea Sediments from HOTRAX

Matthew Zechmeister, University of Oklahoma, Paleomagnetic and rock-magnetic study of Lower Carboniferous carbonates in NW Montana and SW Alberta: Testing for a strain related orogenic remagnetization
pole moments... are associated with individual atoms, but

\[ q_s = -K_s \nabla \tilde{h} \quad \text{(Darcy’s Law)} \]

\[ q_s = -\kappa \nabla \tilde{T} \quad \text{(Fourier’s Law)} \]

\[ j = \sigma \tilde{E} = -\sigma \nabla \tilde{V} \quad \text{(Ohm’s Law)} \]

\[ B = \mu \tilde{H} = -\mu \nabla \tilde{\phi} \]

Exhibit A. Conductive flux in isotropic media - Darcy [1856]; Fourier [1822]; Ohm [1827]; Maxwell [1873]. The quantity on the left-hand side is called flux for groundwater \( q_s \) [m\(^3\) m\(^{-2}\) s\(^{-1}\)] or \( q_s \) [m s\(^{-1}\)] and for heat \( q_s \) [J m\(^{-2}\) s\(^{-1}\)] or \([W m^{-2}]\); current density for electricity \( j \) [C m\(^{-2}\) s\(^{-1}\)] or \([A m^{-2}]\); and flux density or induction for magnetism \( B \) [Wb m\(^{-2}\)] or \([T]\).

Magnetic flux can be conceived in a manner analogous to these other fluxes, albeit with some fundamental differences. Maxwell [art 428] wrote “The problem of induced magnetism, when considered with respect to the relation between magnetic induction and magnetic force, corresponds exactly with the problem of the conduction of electric currents through heterogeneous media... The magnetic force is derived from the magnetic potential precisely as the electric force is derived from the electric potential. The magnetic induction is a quantity of the nature of a flux, and satisfies the same conditions of continuity as the electric current does. In isotropic media the magnetic induction depends on the magnetic force in a manner which exactly corresponds with that in which electric current depends on the electromotive force. The specific magnetic inductive capacity in the one problem corresponds to the specific conductivity in the other. Hence Thomson, in his Theory of Induced Magnetism [1872], has called this the permeability of the medium.”

This analogy is satisfying to our physical intuition, but surprisingly, over a century later there is still broad disagreement over its basic validity, or more generally over the fundamental physical meaning of the fields \( B \) (magnetic flux density or magnetic induction) and \( H \) (magnetic field intensity or magnetizing force) [e.g., Crangle & Gibbs, 1994; Carpenter, 1999]. On the one hand, in the Feynman Lectures on Physics [1964] we read “\( B \) and \( E \) are physically the fundamental fields, and \( H \) is a derived idea”, Maxwell’s equations are written without reference to \( H \), which is only introduced as a mathematical convenience for discussing magnetism and matter. Similarly, Crangle & Gibbs [1994] advocate dropping \( H \) from the magnetic vocabulary. On the other hand, in Stacey & Banerjee [1974] we read “the MKS system has suffered two major disadvantages. Firstly in treating magnetism it was usually presented in a manner which implied that \( B \) not \( H \) was fundamental. Fortunately this fallacy has now been refuted by experimental evidence [Whitworth & Stopes-Roe, 1971].” Dunlop & Özdemir [1997] also dispute the primacy of \( B \). “Microscopic di-

Scalar Potential and Lines of Force

In each of the flux laws in Exhibit A, flow is driven by differences in a scalar potential or analogous quantity. Potential is closely related to, and defined in terms of, potential energy in relation to a field source. “The (electrostatic) Potential at a Point is ... the work which must be done by an external agent in order to bring the unit of positive electricity from an infinite distance (or any other point where the potential is zero) to the given point” [Maxwell, art 70]. Electrical potential is therefore measured in \([J/C]\) (i.e. \([V]\)), and gravitational potential in \([J/kg]\). A magnetostatic scalar potential can be similarly defined in terms of a point monopole source and a unit test monopole, and Coulomb’s experiments showed that “isolated” poles (at the ends of magnetic needles) interact according to the same laws as electric charges. (We’ll get to dipoles and currents in a moment.

The corresponding vector fields (negative gradients of the scalar potentials) are force fields (\([N/C]\) or \([V/m]\)) for \( E \), \([N/kg]\) for \( g \); note that the SI units of \( H \) \([A/m]\) show that magnetic potential must be measured in \([A]\). “In every part of the course of a line of force, it is proceeding from a place of higher potential to a place of lower potential. Hence a line of force cannot return unto itself, but must have a beginning and an end. The beginning of a line of force must be in a positively charged surface, and the end... must be in a negatively charged surface” [Maxwell, art 82]. Such an arrangement is shown in Figure 1 for two equal and opposite charges or poles. The same graph could equally well depict the isotherms and \( T \)-gradient vectors around a heat source and sink, or their hydraulic equivalents: “Potential, in electrical science, has the same relation to Electricity that Pressure, in Hydrostatics, has to Fluid, or that Temperature, in Thermodynamics, has to Heat. Electricity, Fluids, and Heat all tend to pass from one place to another, if the Potential, Pressure or Temperature is greater in the first place than in the second.” [Maxwell, Art. 72].

In isotropic media the flow lines generally coincide with these lines of force, so we can also interpret Figure 1 as showing flow lines for heat, groundwater or electric current, but not for magnetism. Because magnetic monopole sources/sinks do not exist, there is no place for magnetic flux lines to begin or end.

\(^5\) William Thomson (Lord Kelvin)

\(^6\) It is always true that \( B = \mu_0 \mu (H+M) \), and for linear isotropic media \( (M=\kappa H) \) with susceptibility \( \kappa \), this reduces to \( B = \mu_0 \mu H \), where permeability \( \mu \) equals \((1+\kappa)\mu_0\).
However, before we continue with the question of flow, there is a twist concerning magnetic potential and lines of force around electric currents.

Electromagnetism changes things in a fundamental way. Ørsted’s famous experiment and subsequent work showed that lines of magnetic force form around a straight filamentary current, and are in fact closed loops, concentric with and perpendicular to the current. Strictly, such closed-loop fields, with nonzero curl, can not be described as the gradient of a scalar potential, nor can such fields be produced by any distribution of static charges or poles. However, following Maxwell, we can conceive such circular field lines to be following the gradient of something like a scalar potential, provided we allow this potential to be a multi-valued function of spatial coordinates around the current. In other words, in following a line of force in one full circuit around the current, we return to our starting point with a higher (or lower) potential than we started with, much like Escher’s Waterfall. “If it were possible to obtain a magnet having only one pole, or poles of unequal strength, such a magnet would be moved round and round the wire continually in one direction. Faraday ... has shewn how to produce the continuous rotation of one pole of a magnet round an electric current by making it possible for one pole to go round and round the current while the other pole does not.” [Maxwell, art 486].

With currents rather than poles as field sources, it makes sense to think of flux driven by differences or spatial derivatives of a vector potential rather than a scalar potential, in particular the derivative that measures rotation: \( \mathbf{B} = \text{curl}(\mathbf{A}) \). In the same way that scalar potential at a point is the sum of potentials due to all “monopolar” sources, the vector potential is obtained by integrating over all elemental current sources \( ids \) (weighted as \( 1/r \)).

The vector potential is a close cousin of the stream function \( \psi \) from fluid dynamics, where the flow velocity of an incompressible fluid is \( \mathbf{v} = \text{curl}(\psi) \). Since flow is now related to the curl rather than the gradient of a potential, flux is parallel rather than perpendicular to the equipotentials (stream function contours).

When current flows in a circular loop rather than in a straight line, the result is very much like a dipole field (Fig. 2), at least at distances that are larger than the loop diameter or pole separation. The “scalar potential” at any point is proportional to the current \( i \) and to the solid angle subtended by the loop at the point:\[\psi = \frac{i \theta}{4\pi} \] “... the potential is in this case a function having an infinite series of values whose common difference is \( 4\pi \). The differential coefficients of the potential have, however, definite and single values at every point.” [Maxwell art 480]. In contrast to the conservative (or irrotational) field of Figure 1, when we move a charge or a pole from one point to another in the solenoidal (or non-divergent) field of Figure 2, the change in potential (i.e., the work done) depends on the path we follow. In the absence of magnetic matter \( \mathbf{B} = \mu_0 \mathbf{H} \), and Figure 2 serves to depict either field.

Near their centers, the potentials and fields of the dipole and the current loop are entirely different. On the equatorial plane of the dipole (Fig. 1) the field vectors are everywhere parallel (let’s say downward). This is not the case for the current loop (Fig. 2): on the equatorial plane (i.e., the plane of the loop), the field vectors are downward outside the loop, but upward inside, antiparallel to the field lines at the dipole center. Magnetic flux may follow solenoidal lines of force like those of Figure 2, but never divergent lines like those of Figure 1.

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\(^8\)Since the curl of a gradient is identically zero.

\(^9\)These ideas are sometimes combined into a complex potential function with real (scalar) and imaginary (vector) components.

\(^10\)Since solid angle is a dimensionless quantity, this gives a scalar potential measured in [A].
Permeability, Magnetization and Flux

Fluid flow according to Darcy’s law and heat flow according to Fourier’s are necessarily irrotational. They may also be non-divergent (at least locally) when there are no sources or sinks, and when the fluid is incompressible. Magnetic flux is necessarily nondivergent and may also be irrotational (at least locally) when there are no free electrical currents. All of these nondivergent flow fields in heterogeneous isotropic media are described in steady state by Laplace’s equation: \( \nabla^2 \text{(potential)} = 0 \). In such cases both our physical intuition related to the perceptible flow of heat or fluids, and the relevant mathematical methods, may be applied directly to magnetic flux.

For example, Figure 3 shows the \( \mathbf{B} \) field in and around a paramagnetic sphere in a previously-uniform external field; it is precisely the same as the steady-state flow field \( \mathbf{q} \) of an incompressible fluid through a high-permeability sphere embedded in a lower-permeability medium with no sources or sinks, and when the fluid is incompressible. Fluid flow according to Darcy’s law and heat flow according to Fourier’s are necessarily irrotational. They may also be non-divergent (at least locally) when there are no sources or sinks, and when the fluid is incompressible. Magnetic flux is necessarily nondivergent and may also be irrotational (at least locally) when there are no free electrical currents. All of these nondivergent flow fields in heterogeneous isotropic media are described in steady state by Laplace’s equation: \( \nabla^2 \text{(potential)} = 0 \). In such cases both our physical intuition related to the perceptible flow of heat or fluids, and the relevant mathematical methods, may be applied directly to magnetic flux.

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Note that the flux distribution in Figure 3 can also be viewed as the superposition of a uniform \( \mathbf{B} \) field (as it would be without the presence of the sphere) and a dipole-like \( \mathbf{B} \) field, similar to that of Figure 2, due to the sphere.

Figure 3. Flux through a high-permeability sphere embedded in a uniform medium. (modified from Knoepfel, 2000)

Figure 4. Flux in an electromagnet. (from Feynman et al., 1964). The flux in an electromagnet (Fig. 4) is often described by analogy with current flow in an electrical circuit. A high-permeability ferromagnetic yoke (pole pieces and conjoining structure) provides the flux path for the magnetic circuit. Currents running through loops of copper wire produce an \( \mathbf{H} \) field that acts as a magnetomotive force, driving magnetic flux through the circuit. In the pole pieces \( \mathbf{B} = \mu \mathbf{H} = \mu_0 (\mathbf{H}_0 + \mathbf{M}) \), and for large \( \mu, \mathbf{M} \gg \mathbf{H}_0 \), so \( \mathbf{B} \) is a little larger than \( \mu_0 \mathbf{M} \). \( \mathbf{B} \) is continuous across the pole faces and into the gap, but \( \mathbf{M} \) and \( \mathbf{H} \) are not. In the low-permeability gap \( \mathbf{M} \) is zero, and \( \mathbf{H} = \mathbf{B} / \mu_0 \approx \mathbf{B} / \mu_0 \gg \mathbf{M} \) is therefore much larger in the gap than it is in the yoke; the corresponding drop in magnetic potential (\( \Delta \phi = \mathbf{H} \cdot \Delta \mathbf{x} \)) across the gap is like the voltage drop across a resistance in an electric circuit, and by analogy the gap is said to have a reluctance\(^2\). In the reluctance gap there is some spreading or “fringing” of the magnetic field, which becomes more significant as size of the gap increases relative to the pole face dimensions. Ampere’s Law in integral form\(^3\) can be written as \( \oint \mathbf{H} \cdot d\mathbf{l} = nI_{\text{enc}}, \text{free} \), where we will integrate over one closed field loop; \( I_{\text{enc}} \) is the free current enclosed by the integration path, and \( n \) is the number of turns through which it flows. This is analogous to Kirchoff’s law, with the sum of resistive/revertant potential drops around the circuit equal to the driving EMF or MMF.

The recently-developed electron-optic technique of off-axis electron holography (e.g., Dunin-Borkowski et al., 1998, 2004; Feinberg et al., 2007) allows direct imaging of magnetic flux in two dimensions. The electron wave transmitted through a TEM specimen undergoes a phase shift (relative to a reference wave transmitted past the specimen); the magnitude of the shift depends on (among other things) the magnetic vector potential \( \mathbf{A} \), specifically the component of \( \mathbf{A} \) parallel to the electron wave propagation direction (normal to the specimen). After correction for other factors the phase shift contours, like stream function contours, show the lines of magnetic flux \( \mathbf{B} \).

The magnetosomes from disrupted cells of magnetotactic bacteria (Fig. 5), like those in intact chains, show high internal flux densities and generally uniform magnetiza-

\(^{2}\)term first used by Oliver Heaviside; SI units are A Wb\(^1\) (or Henry\(^4\))

\(^{3}\) (and neglecting the displacement term)

cont’d. on p. 10...
tion, as well as strong continuity of flux from one particle to the next, with only minimal fringing. Note that here, unlike Figures 3 and 4, there is no applied external field, nor are there free electrical currents, so if we are to think of the flux as being driven by a magnetomotive force, that MMF must originate in the remanent magnetization of the particles.

We’ll return to remanence in a bit, but first, we cannot write about magnetic flux without at least a brief appreciation of electromagnetic induction.

Induction

Electric currents produce magnetic fields. Faraday searched tirelessly for evidence of a reciprocal effect, ultimately discovering that changing magnetic fields produce transient electric currents, the phenomenon of magnetic induction. It is not by mere coincidence that the $B$ field is also commonly called the “induction.”

Maxwell was very concerned with the getting at the fundamental physical meaning of the electromagnetic fields; he followed Ampère in associating $B$ with electrical currents, and thus in a general way with kinetic energy. Specifically, Maxwell defined the vector potential $A$ as a vector of electromagnetic or electrokinetic momentum. Since $B = \text{curl}(A)$, changes in $B$ with time $dB/dt$ are equivalent to changes in the circulating electrokinetic momentum $\text{curl}(dA/dt)$. In mechanics, the time derivative of momentum is a force, and by extension the changing electrokinetic momentum is equivalent to an electromotive force. Thus $dB/dt = \text{curl}(E)$, which is Faraday’s law of induction.

Most of our magnetometers and susceptometers operate on this principle. The magnetic flux through a sensing coil, due to the magnetization of a sample, can be made to vary with time by displacing or rotating the sample, or by varying the applied field. The EMF acting on the coil is

$$\oint_{\text{coil}} E \cdot ds = \int_{\text{coil}} (\nabla \times E) \cdot dA$$

$$= \frac{d}{dt} \int_{\text{coil}} (\vec{B} \cdot \hat{n}) \cdot dA = d\Phi / dt$$

where $\Phi$ is the total flux through the area enclosed by the coil. This voltage depends on $B$ rather than $H$, and therefore it can be greatly amplified by placement of a high-permeability (e.g., ferrite) core in the coil.

Ferromagnetism, $B$ and $H$

In ferromagnetic materials we have magnetization in the absence of applied fields or free electrical currents. Figure 6 shows the two prevalent fundamental ways of thinking about the $B$ and $H$ fields inside a permanent magnet with a uniform magnetization $M$. The pole-based view, depicted on the right-hand side, is based on a direct analogy with electrostatics: the divergence of $M$ (like that of polarization $P$ across the surface gives a nonzero density $\sigma_M$ of “bound poles” (cf. bound charges) there. These poles in turn produce internal and external scalar potentials and $H$ fields exactly as in Figure 1, and by this conception $B$ inside the material is a derived or secondary quantity, obtained by vector summing of $M$ and $H$. Note however that if we still wish to consider $H$ as a magnetizing force and $B$ as a resultant flux, $B = \mu H$ paradoxically requires $\mu < 0$ for permanent magnets, since $B$ and $H$ are antiparallel inside.

The alternative view (left side) treats $M$ as arising from elemental atomic-scale current loops. In integrating over a uniformly-magnetized volume, all internal currents cancel and the total net effective magnetization current runs only along the surface, following lines of latitude, with current density $= \text{curl}(M)$. These Ampérian “bound currents” are no more directly perceptible than the “bound poles” on the right side; they produce vector potentials and a $B$ field as in Fig 2. For the spherical case we are considering, $B$ inside the magnet is $2M/3$. By this approach, $H$ inside the material is a derived or secondary quantity defined as $H = B/\mu_0 M$.

Whether we prefer to think in terms of poles or currents is to some extent irrelevant: the internal and external distribution of fields is identical for the two calculations. Because of this “equivalence principle”, according to Carpenter [1999], it is hopeless to attempt experimentally to distinguish between the pole-based and current-based models (and thereby determine which is the fundamental field) by measuring external forces or torques on permanent magnets or steady current loops; he attributes the result of Whitworth & Stopes-Roe [1971] to shortcomings in experimental technique.

Are there other ways to distinguish between these alternatives experimentally? Einstein considered it possible and important to do so: “two entirely different ways in which a magnetic field can be produced...could hardly be considered as satisfactory. It appears ...that as much...”

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1Thomas Simpson elaborates on this in his eloquent little book, Figures of Thought: A Literary Appreciation of Maxwell’s Treatise on Electricity and Magnetism.

1This antiparallel $H$ is the “demagnetizing field”, which is the key to understanding shape anisotropy.
may be said in favour of Ampère’s hypothesis\textsuperscript{16} as against it and that the question concerns important physical principles. We have therefore made the experiments here to be described, by which we have been able to show that the magnetic moment of an iron molecule is really due to a circulation of electrons.” [Einstein & de Haas, 1915]. Their observation of a periodic torque on an iron cylinder suspended vertically in a slowly-alternating vertical saturating field demonstrated that magnetism is fundamentally related to angular momentum at some scale; the modern quantum mechanical interpretation involves electron spin rather than circulation.

Feynman tells us that despite the apparent analogy of the fields $\mathbf{E}$ and $\mathbf{H}$ in Exhibit A, a deeper identity of $\mathbf{E}$ and $\mathbf{B}$ comes from relativity. Imagine an electron in motion parallel to a wire carrying electrical current. The $\mathbf{B}$ field due to the current exerts a Lorentz force on the moving electron: $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$. For thought-experiment purposes we can also specify that the velocity of the single electron is the same as those of the conduction electrons in the wire. In the inertial reference frame in which the wire is at rest there is a $\mathbf{B}$ field but no $\mathbf{E}$ field. A different reference frame can be defined such that the wire is in motion and the electron is at rest; here the $\mathbf{B}$ field vanishes and there is only an $\mathbf{E}$ field (see Feynman for relativistic details). Clearly, therefore, there is only a single electromagnetic field at work, and it manifests itself as $\mathbf{B}$ or $\mathbf{E}$ or some combination according to which inertial reference frame we are observing in.

Arguments about the fundamental significance of $\mathbf{B}$ and $\mathbf{H}$ “have tended to drop out of sight as lacking fruitful result rather than as the emergence of any consensus”, according to Carpenter [1999], who works through the complete development of potentials and vectors for two universes: one called U in which potentials $\phi$ and $\mathcal{A}$ arise exclusively from stationary and moving charges, and poles are fictitious equivalent sources; and another one called $U'$ in which poles are the true sources and charges are fictitious equivalents for generating potentials $\phi'$ and $\mathcal{A}'$. The four field vectors $\mathbf{E}, \mathbf{B}, \mathbf{D}$ and $\mathbf{H}$ in U have perfectly symmetrical equivalents $\mathbf{H}', \mathbf{D}', \mathbf{B}'$ and $\mathbf{E}'$, respectively in $U'$. Concepts like magnetomotive force “can coexist with that of voltage only in a universe defined as $U + U'$, requiring four different primary field vectors, not two.”

Which universe do we live in? Do we have a choice? What is the ultimate, fundamental, real physical meaning of these vectors? Are magnetic bound poles less “real” than electrical bound charges or Ampérian bound magnetization currents? Should $\mathbf{H}$ be banished from physics, except as a mathematical convenience? What is this thing called flux? We leave these questions as an exercise for the student...

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\textbf{Jean Baptiste Joseph Fourier}

b. 21 March 1768, Auxerre, France

d. 16 May 1830, Paris, France

Best known as a mathematician and physicist, Fourier was also an accomplished Egyptologist and administrator. In 1798, he accompanied Napoleon to Egypt, where he served as governor of Lower Egypt. After returning to France, he played a major role in the publication of the massive \textit{Description de l’Égypte}. While serving as Prefect of the Department of Isère, Fourier began his famous work on heat conduction. In 1807, he publicly delivered his memoir \textit{On the Propagation of Heat in Solid Bodies} which was never published, in part due to objections by Laplace and Lagrange over the controversial use of expansions of functions as trigonometric series. Now known as Fourier Series, the expansions were ultimately incorporated in his famous \textit{Théorie analytique de la chaleur} (\textit{Analytical theory of heat}, published in 1822. This work (often described as “a great mathematical poem”) also significantly advanced the understanding and application of boundary-value problems and the theory of functions of a real variable. Fourier also made studies on the heat budget of planetary bodies and is credited as being the first to recognize that atmospheric gases can trap heat and help warm the Earth. Fourier was elected to the Académie des Sciences in 1817 and to the Académie Française in 1826.
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